Group Representations

F - field, V - vector space over F

$\Longrightarrow \quad GL(V) = \{T : V \to V \text{ linear bijection}\}$

e.g.: $V = F^n$, $GL_n(F) := GL(F^n)$

$$GL(F^n) \subseteq \mathrm{Mat}_{n\times n}(F)$$

G - group; V = vector space$/_{F}$

a representation of G in V is a homomorphism $\varphi : G \longrightarrow GL(V)$

Write: $g \in G$, $\varphi_g = \varphi(g) \in GL(V)$

If $V = F^n$, $\varphi : G \longrightarrow GL_n(F)$ is called a matrix representation of G

We will mostly consider $F = \mathbb{C}$

Rem: $\mathbb{R} \subseteq \mathbb{C}$ subfield,

$$G \xrightarrow{\varphi} GL_n(\mathbb{R}) \leq GL_n(\mathbb{C})$$

The dimension (or degree) of $\varphi : G \longrightarrow GL(V)$ is $\dim_F V$

We will look only at finite dimensional representations